SB/F/06-339

Total Quantum Zeno effect and Intelligent States for a two level system in a squeezed bath

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In this work we show that by frequent measurements of adequately chosen observables, a complete suppression of the decay in an exponentially decaying two level system interacting with a squeezed bath is obtained. The observables for which the effect is observed depend on the the squeezing parameters of the bath. The initial states which display Total Zeno Effect are intelligent states of two conjugate observables associated to the electromagnetic fluctuations of the bath.

arXiv:quant-ph/0610179 v1 20 Oct 2006

I. INTRODUCTION

Frequent measurements modify the dynamics of a quantum system. This result of quantum measurement theory is known as the quantum Zeno effect(QZE)[1, 2, 3, 4, 5] and has attracted much attention since first discussed. The QZE is related to the suppression of induced transitions in interacting systems or the reduction of the decay rate in unstable systems. It has been also pointed out that the opposite effect, i.e the acceleration of the decay process, maybe caused by frequent measurement too and this effect is known as Anti-Zeno effect(AZE). The experimental observation of QZE in the early days was restricted to oscillating quantum systems [6] but recently, both QZE and AZE were successfully observed in irreversible decaying processes.[7, 8, 9].

The quantum theory of measurement predicts a reduction in the decay rate of an unstable system if the time between successive measurements is smaller than the Zeno time which in general is smaller than the correlation time of the bath. The effect is universal in the sense that it does not depend on the measured observable whenever the time between measurements is very small. This observation does not preclude the manifestation of the Zeno effect for larger times (for example in the exponentially decaying regime) for well selected observables in a particular bath.

Recently it was shown in Ref.([10]) that reduction of the decay rate occurs in an exponentially decaying two level system when interacting with a squeezed bath. In this case variations on the squeezing phase of the bath may lead to the appearance of either Zeno or anti-Zeno effect when continuously monitoring the associated fictitious spin. In this work we show that for the same system it is possible to select a couple observables whose measurement for adequately prepared systems lead to the total suppression of the transitions i.e Total Zeno Effect.

This paper is organized as follows: In section (II) we discuss some general facts and review some results obtained in reference [10] which are needed for our discussion. In Section (III) we define the system we deal with and identify the observables and the corresponding initial states which are shown to display Total Zeno Effect. In section (IV) we show that the initial states which show Total Zeno Effect are intelligent spin states, i.e states that saturate the Heisenberg Uncertainty Relation between the (fictitious) spin operators. Finally, we discuss the results in Section (V).

II. TOTAL ZENO EFFECT IN UNSTABLE SYSTEMS

Consider a closed system with Hamiltonian H and an observable A with discrete spectrum. If the system is initialized in an eigenstate $|a_n\rangle$ of A with eigenvalue a_n, the probability of survival in a sequence of S measurements, that is the probability that in all measurements one gets the same result a_n, is

$$P_n(\Delta t, S) = \left(1 - \frac{\Delta t^2}{\hbar^2}\Delta_n^2 H\right)^S \qquad (1)$$

where

$$\Delta_n^2 H = \langle a_n|H^2|a_n\rangle - \langle a_n|H|a_n\rangle^2 \qquad (2)$$

and Δt is the time between consecutive measurements. In the limit of continuous monitoring ($S \to \infty, \Delta t \to 0$ and $S\Delta t \to t$), $P_n \to 1$ and the system is freezed in the initial state.

For an unstable system and considering evolution times larger than the correlation time of the reservoir with which it interacts, the irreversible evolution is well described in terms of the Liouville operator $L\{\rho\}$ by the master equation

$$\frac{\partial \rho}{\partial t} = L\{\rho\} \ . \qquad (3)$$

If measurements are done frequently, the master equation (3) is modified. The survival probability is time dependent and is shown to be given by [10]

$$P_n(t) = \exp\left\{\langle a_n|L\{|a_n\rangle\langle a_n|\}|a_n\rangle t\right\} \quad . \qquad (4)$$

This expression is valid only when the time between consecutive measurements is small enough but greater than the correlation time of the bath. For mathematical simplicity in what follows we consider the zero correlation time limit for the bath and then one is allowed to take the limit of continuous monitoring. From equation (4) one observe that the Total Zeno Effect is possible when

$$\langle a_n|L\{|a_n\rangle\langle a_n|\}|a_n\rangle = 0 \quad . \tag{5}$$

Then, for times larger than the correlation time, the possibility of having Total Zeno effect depends on the dynamics of the system (determined by the interaction with the baths), on the particular observable to be measured and on the initial state.

The concept of zero correlation time of the bath τ_D is of course an idealization. If this time is not zero, the equation (4) is only approximate, since Δt cannot be strictly zero and at the same time be larger than τ_D. Also, if equation (5) is satisfied, then equation (4) must be corrected, taking the next non-zero contribution in the expansion of $\rho(\Delta t)$.

In that case, a decay rate proportional to Δt appears, and the decay time is $\propto \frac{1}{\gamma^2 \Delta t}$, which is in general a number much larger than the typical evolution time of the system. Notice that as the spectrum of the squeezed bath gets broader, τ_D becomes smaller, and one is able to choose a smaller Δt, approaching in this way, the ideal situation and the Total Zeno Effect.

III. TOTAL ZENO OBSERVABLES

In the interaction picture the Liouville operator for a two level system in a broadband squeezed vacuum has the following structure [11],

$$\begin{aligned} L\{\rho\} &= \frac{1}{2}\gamma\,(N+1)\,(2\sigma_-\rho\sigma_+ - \sigma_+\sigma_-\rho - \rho\sigma_+\sigma_-) \\ & \frac{1}{2}\gamma N\,(2\sigma_+\rho\sigma_- - \sigma_-\sigma_+\rho - \rho\sigma_-\sigma_+) \\ & -\gamma M e^{i\phi}\sigma_+\rho\sigma_+ - \gamma M e^{-i\phi}\sigma_-\rho\sigma_- \quad . \end{aligned} \tag{6}$$

whereh γ is the vacuum decay constant and $N, M = \sqrt{N(N+1)}$ and ψ are the parameters of the squeezed bath. Here σ_- and σ_+ are the two ladder operators,

$$\sigma_+ = \frac{1}{2}(\sigma_x + i\sigma_y) \qquad \sigma_- = \frac{1}{2}(\sigma_x + i\sigma_y) \tag{7}$$

with σ_x,σ_y and σ_z the Pauli matrices.

Let us introduce the Bloch representation of the two level density matrix

$$\rho = \frac{1}{2}\,(1 + \vec{\rho}\cdot\vec{\sigma}) \tag{8}$$

In this representation the master equation takes the form:

$$\begin{aligned} \frac{\partial\rho}{\partial t} &= -\frac{1}{2}\gamma\,(N+1)\left((1+\rho_z)\sigma_z + \frac{1}{2}\rho_x\sigma_x + \frac{1}{2}\rho_y\sigma_y\right) \\ & +\frac{1}{2}\gamma N\left((1-\rho_z)\sigma_z - \frac{1}{2}\rho_x\sigma_x - \frac{1}{2}\rho_y\sigma_y\right) \\ & -\frac{1}{2}\gamma M\rho_x(\cos(\psi)\sigma_x - \sin(\psi)\sigma_y) \\ & +\frac{1}{2}\gamma M\rho_y(\sin(\psi)\sigma_x + \cos(\psi)\sigma_y) \end{aligned} \tag{9}$$

and has the following solutions for Bloch vector components that give the behavior of the system without measurements

$$\begin{aligned} \rho_x(t) &= \left(\rho_x(0)\sin^2(\psi/2) + \rho_y(0)\sin(\psi/2)\cos(\psi/2)\right) \\ & e^{-\gamma(N+1/2-M)\,t} \\ & + \left(\rho_x(0)\cos^2(\psi/2) - \rho_y(0)\sin(\psi/2)\cos(\psi/2)\right) \\ & e^{-\gamma(N+1/2+M)\,t} \end{aligned} \tag{10}$$

$$\begin{aligned} \rho_y(t) &= \left(\rho_y(0)\cos^2(\psi/2) + \rho_x(0)\sin(\psi/2)\cos(\psi/2)\right) \\ & e^{-\gamma(N+1/2-M)\,t} \\ & + \left(\rho_y(0)\sin^2(\psi/2) - \rho_x(0)\sin(\phi/2)\cos(\psi/2)\right) \\ & e^{-\gamma(N+1/2+M)\,t} \end{aligned} \tag{11}$$

$$\rho_z(t) = \rho_z(0)e^{-\gamma(2N+1)t} + \frac{1}{2N+1}\left(e^{-\gamma(2N+1)t} - 1\right) \tag{12}$$

Now consider the hermitian operator σ_μ associated to the fictitious spin component in the direction of the unitary vector $\hat{\mu} = (\cos(\phi)\sin(\theta), \sin(\phi)\sin(\theta), \cos(\theta))$ defined by the angles θ and ϕ,

$$\sigma_\mu = \vec{\sigma}\cdot\hat{\mu} = \sigma_x\cos(\phi)\sin(\theta) + \sigma_y\sin(\phi)\sin(\theta) + \sigma_z\cos(\theta) \tag{13}$$

The eigenstates of σ_μ are,

$$|+\rangle_\mu = \cos(\theta/2)\,|+\rangle + \sin(\theta/2)\,\exp\,(i\phi)\,|-\rangle \tag{14}$$

$$|-\rangle_\mu = -\sin(\theta/2)\,|+\rangle + \cos(\theta/2)\,\exp\,(i\phi)\,|-\rangle \tag{15}$$

If the system is initialized in the state $|+\rangle_\mu$ the survival probability at time t is

$$P^+_\mu(t) = \exp\{F(\theta,\phi)\;t\} \tag{16}$$

where

$$F(\theta,\phi) = {}_\mu\langle +|\;L\,\{\;|+\rangle_\mu\,{}_\mu\langle +|\;\}\;|+\rangle_\mu \quad . \tag{17}$$

In this case the function $F(\theta,\phi)$ has the structure

$$\begin{aligned} F(\theta,\phi) &= -\frac{1}{2}\gamma\,(N+1)\left(\rho_z(0)+\rho_z^2(0)+\frac{1}{2}\rho_x^2(0)+\frac{1}{2}\rho_y^2(0)\right) \\ &+\frac{1}{2}\gamma N\left((\rho_z(0)-\rho_z^2(0)-\frac{1}{2}\rho_x^2(0)-\frac{1}{2}\rho_y^2(0)\right) \\ &-\frac{1}{2}\gamma M\rho_x(0)(\cos(\psi)\rho_x(0)-\sin(\psi)\rho_y(0)) \\ &+\frac{1}{2}\gamma M\rho_y(0)(\sin(\psi)\rho_x(0)+\cos(\psi)\rho_y(0)) \qquad (18) \end{aligned}$$

where now $\vec{\rho}(0)=\hat{\mu}$ is a function of the angles..

In figure (1) we show $F(\phi,\theta)$ for $N=1$ and $\psi=0$ as function of ϕ and θ. The maxima correspond to $F(\phi,\theta)=0$. For arbitrary values of N and ψ there are two maxima corresponding to the following angles:

$$\phi_1^M=\frac{\pi-\psi}{2} \quad \text{and} \quad \cos(\theta^M)=-\frac{1}{2\,(N+M+1/2)} \qquad (19)$$

and

$$\phi_2^M=\frac{\pi-\psi}{2}+\pi \quad \text{and} \quad \cos(\theta^M)=-\frac{1}{2\,(N+M+1/2)} \qquad (20)$$

These preferential directions given by the vectors $\hat{\boldsymbol{\mu}}_{\mathbf{1}}=$

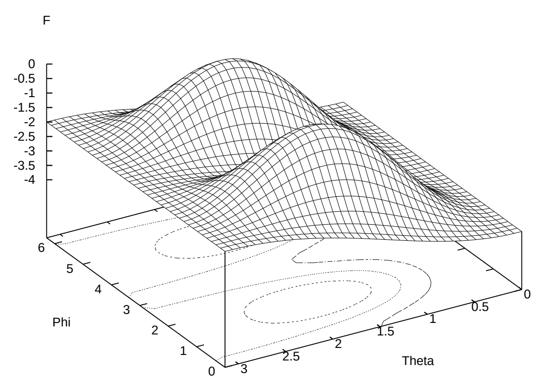

FIG. 1: $F(\phi,\theta)$ for $N=1$ and $\psi=0$

$(\cos(\phi_1^M)\sin(\theta^M),\sin(\phi_1^M)\sin(\theta^M),\cos(\theta^M))$ and $\hat{\boldsymbol{\mu}}_{\mathbf{2}}=(\cos(\phi_2^M)\sin(\theta^M),\sin(\phi_2^M)\sin(\theta^M),\cos(\theta^M)))$ define the operators σ_{μ_1} and σ_{μ_2} which show Total Zeno Effect if the initial state of the system is the eigenstate $|+\rangle_{\mu_1}$ or respectively $|+\rangle_{\mu_2}$.

To be specific let us consider measurements of the observable $\sigma_{\mu_1}=\vec{\sigma}\cdot\hat{\mu}_1$ (analogous results are obtained for σ_{μ_2}). The modified master equation with measurements of σ_{μ_1} is given by [10],

$$\frac{\partial\rho}{\partial t}=P_{\mu_1}^+\,L\left\{\rho\right\}\,P_{\mu_1}^+ +\left(1-P_{\mu_1}^+\right)\,L\left\{\rho\right\}\,\left(1-P_{\mu_1}^+\right) \qquad (21)$$

where

$$P_{\mu_1}^+=|+\rangle_{\mu_1\;\mu_1}\langle+| \qquad (22)$$

and $L\{\rho\}$ is given by (6).

Besides of the Total Zeno effect obtained in the cases specified above it is also very interesting to discuss the effect of measurements for other choices of the initial state. This can be done numerically. In figure (2) we show the evolution of $\langle\sigma_{\mu_1}\rangle$, that is the mean value of observable σ_{μ_1}, when the system is initialized in the state $|+\rangle_{\mu_1}$ without measurements (master equation (6)) and with frequent monitoring of σ_{μ_1} (master equation (21)). Consistently with our discussion of frequent measurements, the system is freezed in the state $|+\rangle_{\mu_1}$ (Total Zeno Effect).

In figure (3) we show the time evolution of $\langle\sigma_{\mu_1}\rangle$ when the initial state is $|-\rangle_{\mu_1}$ without measurements and with measurements of the same observable as in previous case. One observes that with measurements the system evolves from $|-\rangle_{\mu_1}$ to $|+\rangle_{\mu_1}$. In general for any initial state the system under frequent measurements evolves to $|+\rangle_{\mu_1}$ which is the stationary state of Eq. (21) whenever we do measurements in σ_{μ_1}. Analogous effects are observed if one measures σ_{μ_2}. In contrast, for measurements in other directions different from those defined by $\hat{\boldsymbol{\mu}}_{\mathbf{1}}$ or $\hat{\boldsymbol{\mu}}_{\mathbf{2}}$, the system evolves to states which are not eigenstates of the measured observables.

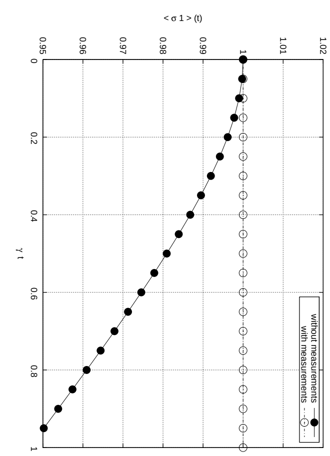

FIG. 2: $\langle\sigma_{\mu_1}(t)\rangle$ for $N=1$ and $\psi=0$. Solid circles: no measurements. Empty circles: with measurements. One measures σ_{μ_1} and the initial state is $|+\rangle_{\mu_1}$

IV. INTELLIGENT STATES

Aragone et al [12] considered well defined angular momentum states that satisfy the equality $(\Delta J_x\Delta J_y)^2=\frac{1}{4}\mid\langle J_z\rangle\mid^2$ in the uncertainty relation. They are called Intelligent States in the literature. The difference with the coherent or squeezed states, associated to harmonic

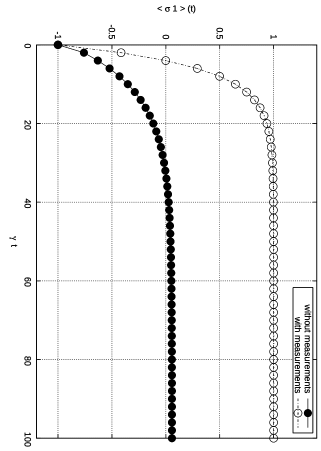

FIG. 3: $\langle\sigma_{\mu_1}(t)\rangle$ for $N = 1$ y $\psi = 0$. Solid circles: no measurements. Empty circles: with measurements. One measures σ_{μ_1} and the initial state is $|-\rangle_{\mu_1}$

oscillators, is that these Intelligent States are not Minimum Uncertainty States (MUS), since the uncertainty is a function of the state itself.

In this section we show that the states $|+\rangle_{\mu_1}$ and $|+\rangle_{\mu_2}$ are intelligent states of two observables associated to the bath fluctuations. The master equation (6) can be written in an explicit Lindblad form

$$\frac{\partial\rho}{\partial t} = \frac{\gamma}{2}\left\{2S\rho S^\dagger - \rho S^\dagger S - S^\dagger S\rho\right\} \tag{23}$$

using only one Lindblad operator S,

$$\begin{aligned} S &= \sqrt{N+1}\sigma_- - \sqrt{N}\exp\{i\psi\}\,\sigma^+ \\ &= \cosh(r)\sigma_- - \sinh(r)\exp\{i\psi\}\,\sigma^+ \end{aligned} \tag{24}$$

Obviously the eigenstates of S satisfy the condition (5). Moreover the states $|\phi_1\rangle = |+\rangle_{\mu_1}$ and $|\phi_2\rangle = |+\rangle_{\mu_2}$ are the two eigenstates of S with eigenvalues $\lambda_\pm = \pm i\sqrt{M}\exp\{i\psi/2\}$.

$$S|\phi_{1,2}\rangle = \lambda_\pm|\phi_{1,2}\rangle \tag{25}$$

Consider now the standard fictitious angular momentum operators for the two level system are $\{J_x = \sigma_x/2, J_y = \sigma_y/2, J_z = \sigma_z/2\}$ and also two rotated operators J_1 and J_2 which are consistent with the electromagnetic bath fluctuations in phase space (see fig. 2 in ref [10]) and which satisfy the same Heisenberg uncertainty relation that J_x and J_y . They are,

$$\begin{aligned} J_1 &= \exp\{i\psi/2J_z\}J_x\exp\{-i\psi/2J_z\} \\ &= \cos(\psi/2)J_x - \sin(\psi/2)J_y \end{aligned} \tag{26}$$

$$\begin{aligned} J_2 &= \exp\{i\psi/2J_z\}J_y\exp\{-i\psi/2J_z\} \\ &= \sin(\psi/2)J_x + \cos(\psi/2)J_y \end{aligned} \tag{27}$$

These two operators are associated respectively with the major and minor axes of the ellipse which represents the fluctuations of bath. Using Eqs. (10) and (11) one can show that the mean values of these operators have the following exponentially decaying evolution:

$$\langle J_1\rangle(t) = \langle J_1\rangle(0)\exp\{-\gamma(N+M+1/2)t\} \tag{28}$$

$$\langle J_2\rangle(t) = \langle J_2\rangle(0)\exp\{-\gamma(N-M+1/2)t\} \tag{29}$$

Notice that the above averages decay with maximum and minimum rates respectively.

In terms of J_1 y J_2 we have

$$J_- = \sigma = (J_x - iJ_y) = \exp\{i\psi/2\}(J_1 - iJ_2)\ , \tag{30}$$

$$J_+ = \sigma^\dagger = (J_x + iJ_y) = \exp\{-i\psi/2\}(J_1 + iJ_2)\ . \tag{31}$$

S has the form

$$S = \exp\{i\psi/2\}\left(\cosh(r) - \sinh(r)\right)\left(J_1 - i\alpha J_2\right) \tag{32}$$

with

$$\alpha = \frac{\cosh(r) + \sinh(r)}{\cosh(r) - \sinh(r)} = \exp\{2r\} \tag{33}$$

Following Rashid *et al (* [13]*)* we define a non hermitian operator $J_-(\alpha)$

$$J_-(\alpha) = \frac{(J_1 - i\alpha J_2)}{(1-\alpha^2)^{1/2}} \tag{34}$$

so that

$$S = \exp\{i\psi/2\}\left(\cosh(r) - \sinh(r)\right)\,(1-\alpha^2)^{1/2}\,J_-(\alpha) \tag{35}$$

After some algebra one obtains then that

$$S = 2\,\lambda_+\,J_-(\alpha) \tag{36}$$

The eigenstates of S are then eigenstates of $J_-(\alpha)$ with eigenvalues $\pm 1/2$. The eigenstates of $J_-(\alpha)$ are are also shown to be intelligent states *i.e* they satisfy the equality condition in the Heisenberg uncertainty relation for J_1 and J_2 .

$$\Delta^2 J_1 \Delta^2 J_2 = \frac{|\langle J_z\rangle|^2}{4} \tag{37}$$

The operator $J_-(\alpha)$ is obtained from J_1 by the following transformation

$$J_-(\alpha) = \exp\{\beta J_z\}J_1\exp\{-\beta J_z\} \tag{38}$$

with

$$\exp\{\beta\} = \sqrt{\frac{1-\alpha}{1+\alpha}} = i\sqrt{\frac{\sinh(r)}{\cosh(r)}} \tag{39}$$

In terms of the real and imaginary parts of $\beta = \beta_r + i\beta_i$,

$$\beta_i = \frac{\pi}{2} \tag{40}$$

$$\exp\{\beta_r\} = \sqrt{\frac{\sinh(r)}{\cosh(r)}} = \left(\frac{N}{N+1}\right)^{1/4} \tag{41}$$

S takes the form,

$$\begin{aligned} S &= 2i\sqrt{M}\exp\{i\psi/2\} \qquad (42)\\ &\exp\{i\frac{\pi}{2}J_z\}\exp\{\beta_r J_z\}\,J_1\,\exp\{-i\frac{\pi}{2}J_z\}\exp\{-\beta_r J_z\} \end{aligned}$$

Finally S may be written in the form:

$$S = 2i\sqrt{M}\exp\{i\psi/2\}U\,J_z\,U^{-1} \tag{43}$$

with

$$U = \exp\{i\frac{\pi}{2}J_z\}\exp\{\beta_r J_z\}\exp\{i\frac{\psi}{2}J_z\}\exp\{-i\frac{\pi}{2}J_y\} \tag{44}$$

Then the eigenstates of S could be obtained from the eigenstates of J_z using U as

$$|\phi_{1,2}\rangle = C_0\;U\;|\pm\rangle \tag{45}$$

where C_0 is a normalization constant. It is quite clear that $|\phi_{1,2}\rangle$ are intelligent states of the observables J_1, J_2,which are rotated versions of J_x, J_y. They are also quasi-intelligent states of the original observables J_x, J_y[13]).

One can verify the above result, by finding directly the eigenstates of S:

$$|\phi_{1,2}\rangle = \sqrt{\frac{N}{N+M}}|+\rangle \pm i\sqrt{\frac{M}{N+M}}e^{-i\psi/2}|-\rangle \tag{46}$$

Finally, when the system is initialized in one of these states, the mean value of J_1 is zero, $\langle J_1\rangle(t) = 0$. Then, the term with the biggest decaying rate does not appear in the mean value of the measured observable σ_μ which becomes:

$$\langle\sigma_\mu\rangle(t) = \langle J_2\rangle(t)\sin(\theta^M) + \langle J_z\rangle(t)\cos(\theta^M) \tag{47}$$

Using the definition of the angle θ^M one can prove that

$$\frac{d\langle\sigma_\mu\rangle}{dt}(0) = 0 \tag{48}$$

which is a neccesary condition in order to obtain Total Zeno Effect when one is measuring the observable σ_μ(See Fig (2)).

V. DISCUSSION

We have shown that Total Zeno Effect is obtained for two particular observables σ_{μ_1} or σ_{μ_2}, for which the azimuthal phases in the fictitious spin representation depend on the phase of the squeezing parameter of the bath and the polar phases depend on the squeeze amplitude r. In this sense, the parameters of the squeezed bath specify some definite atomic directions.

When performing frequent measurements on σ_{μ_1}, starting from the initial state $|+\rangle_{\mu_1}$, the system freezes at the initial state as opposed to the usual decay when no measurements are done. On the other hand, if the system is initially prepared in the state $|-\rangle_{\mu_1}$, the frequent measurements on σ_{μ_1} will makes it evolve from the state $|-\rangle_{\mu_1}$ to $|+\rangle_{\mu_1}$. More generally, when performing the measurements on σ_{μ_1}, any initial state evolves to the same state $|+\rangle_{\mu_1}$ which is the steady state of the master equation (21) in this situation.

The above discussion could appear at a first sight surprising. However, taking a more familiar case of a two-level atom in contact with a thermal bath at zero temperature, if one starts from any initial state, the atom will necessarily decay to the ground state. This is because the time evolution of $\langle\sigma_z\rangle$ is the same with or without measurements of σ_z. In both cases the system goes to the ground state, which is an eigenstate of the measured observable σ_z. In the limit $N, M \to 0$, $\sigma_{\mu_1} \to -\sigma_z$, and the state $|+\rangle_{\mu_1} \to |-\rangle_z$, which agrees with the known results.

Finally, we also found that the eigenstates of S are also quasi-intelligent states of the observables J_x, J_y *i.e* intelligent states of the rotated version of the observables, that is, of J_1, J_2. Starting from an eigenstate of σ_z, these intelligent states are obtained by applying the transformation defined by U.

A. Acknowledgements

Two of the authors(D.M. and J.S.) were supported by Did-Usb Grant Gid-30 and by Fonacit Grant No G-2001000712.

M.O was supported by Fondecyt # 1051062 and Nucleo Milenio ICM(P02-049)

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